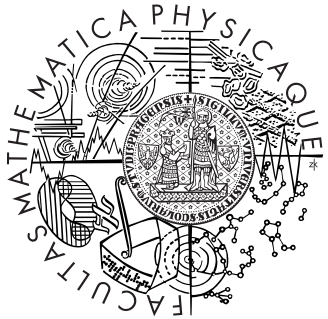
Extending Partial Representations of Subclasses of Chordal Graphs

Pavel Klavík

joint work with J. Kratochvíl, Y. Otachi and T. Saitoh



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Japan Advanced Institute of Science and Technology,
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Graduate School of Engineering, Kobe University,
Kobe, Japan

ISAAC 2012, Taipei

We study a generalization of the RECOGNITION problem called the PARTIAL REPRESENTATION EXTENSION problem.

For example for planar graphs:

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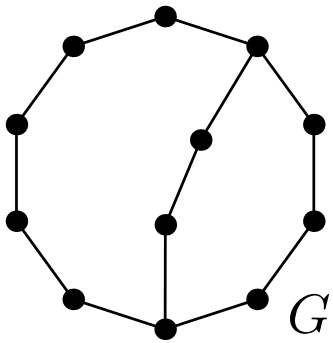

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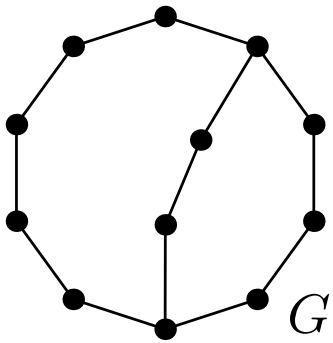

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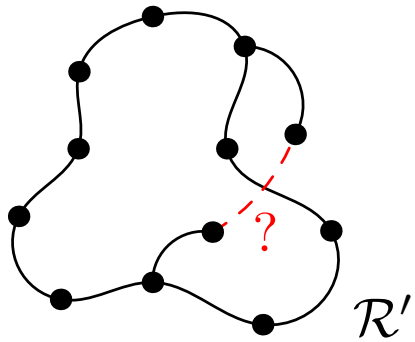

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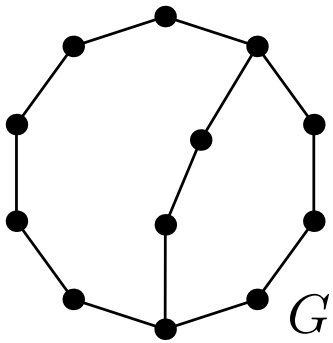

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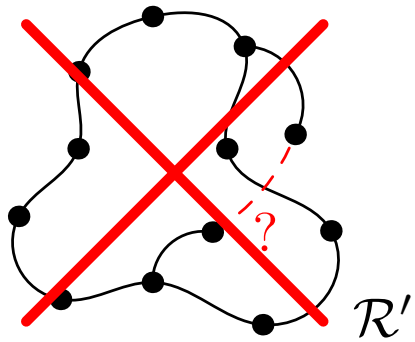

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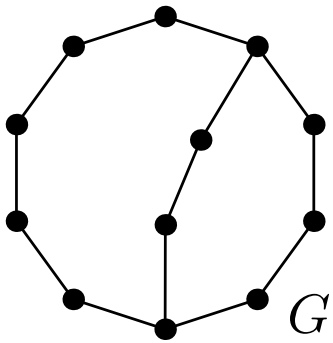

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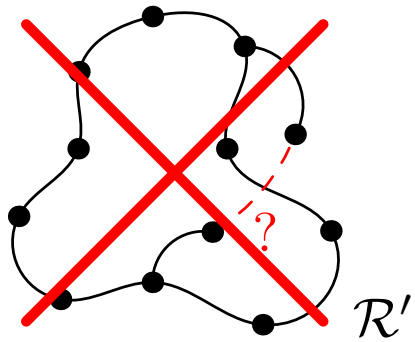

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We study intersection representations:

$$\mathcal{R} = \{R_v : v \in V\} \quad \text{such that} \quad uv \in E \iff R_u \cap R_v \neq \emptyset.$$

When we restrict the sets R_v, we get some nice classes:

INT: $K_{1,3}$

PINT/UINT: K_4^-

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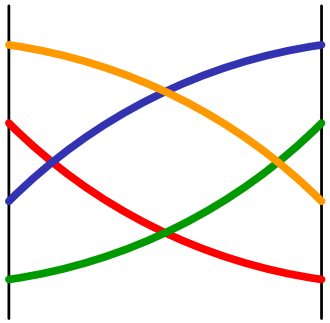

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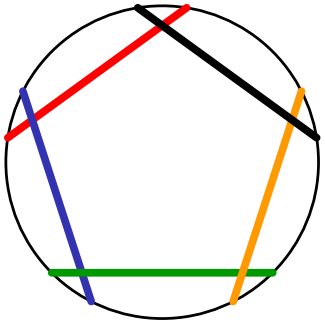

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- A representation $\mathcal{R}$ extends $\mathcal{R}'$ if it assigns the same sets to G' as $\mathcal{R}'$.

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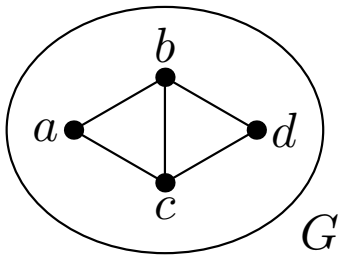
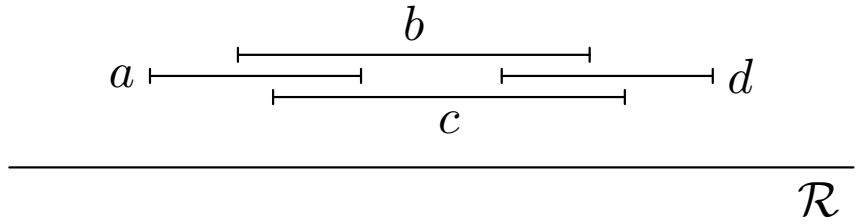
$G \xrightarrow{?} \mathcal{R}$





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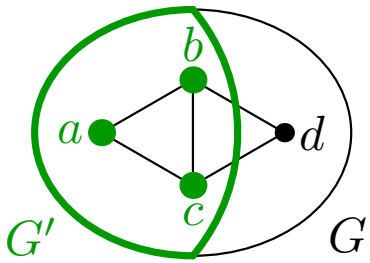

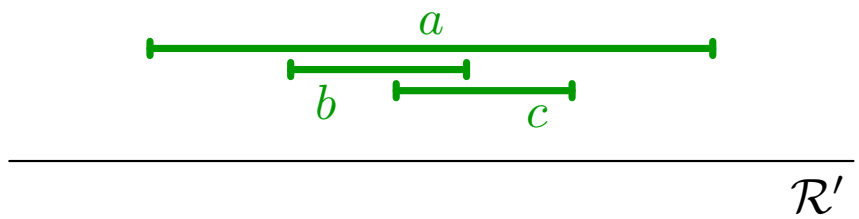

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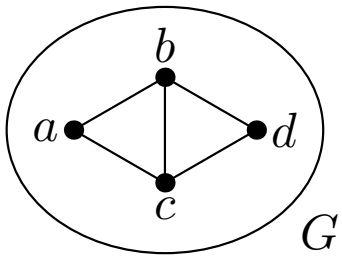

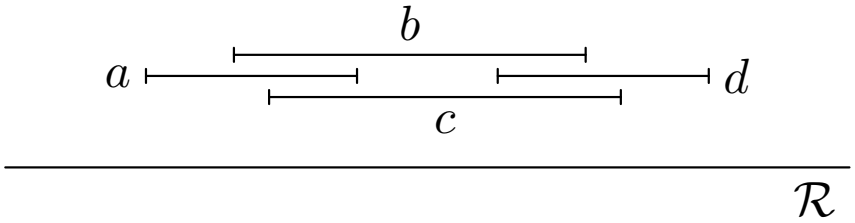

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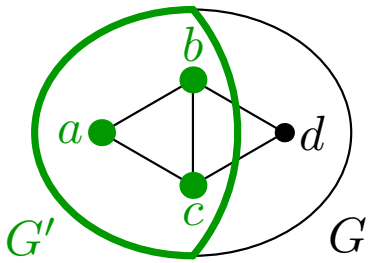

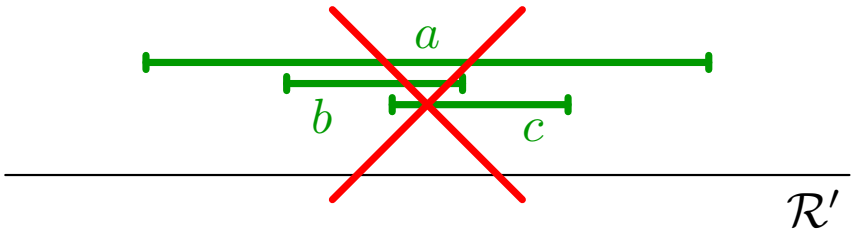

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- Planar graphs: Angelini et al. (SODA 2010), Patrignani (2006).
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Motivation for the partial representation extension problem:

1. The problem is very natural and applicable.
2. Similar types of extension problems are frequently studied:
   - For example a very hard pre-coloring extension problem.
3. The problem forces better understanding of studied classes:
   - For interval graphs or function graphs, we can use known structure.
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Extending chordal graphs

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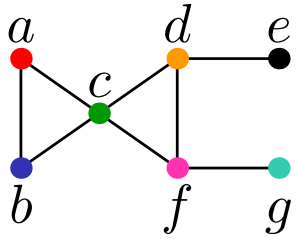

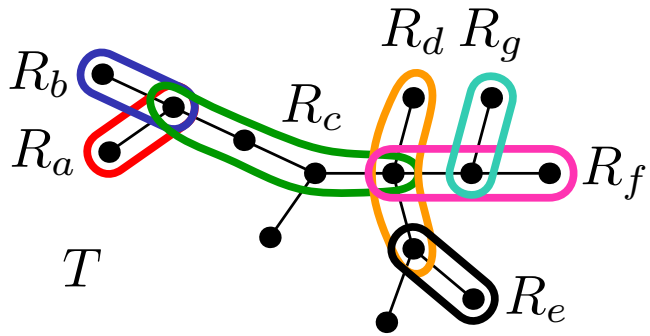

We also consider three subclasses of CHOR:

- PATH: Every subtree is a path.
- INT: The tree T is a path.
- PINT: No subpath is a proper subset of another subpath, i.e.,

$$R_u \subseteq R_v \implies R_u = R_v.$$

$$\text{PINT} \subsetneq \text{INT} \subsetneq \text{PATH} \subsetneq \text{CHOR}.$$

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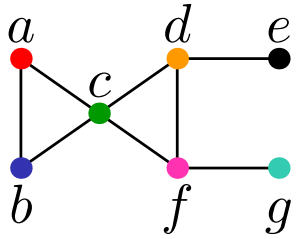

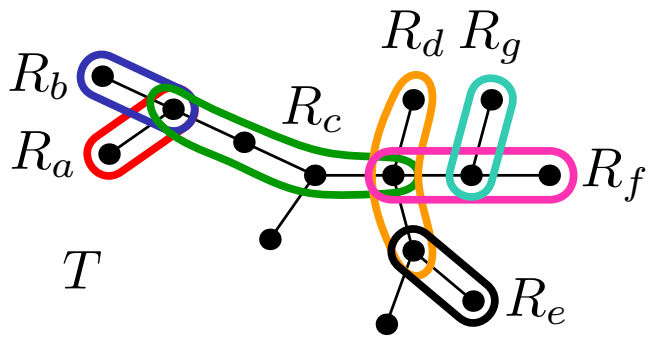

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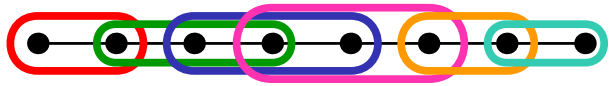

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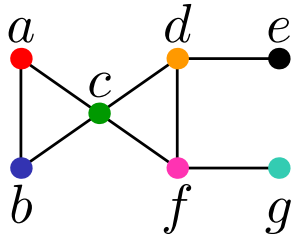

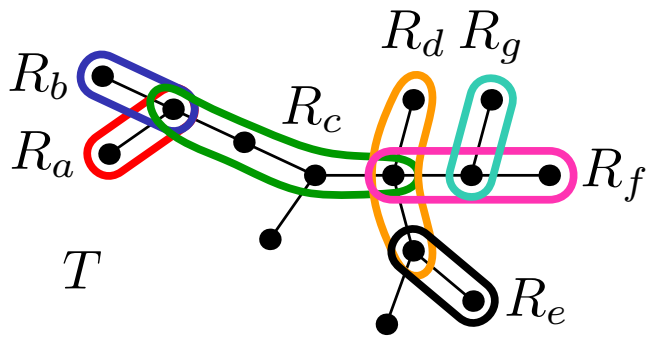

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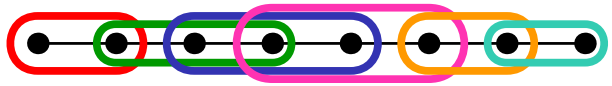

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What is a partial representation of a chordal graph?

$\mathcal{R}'$ gives some tree T' which is modified in $\mathcal{R}$ to T:

- FIXED: the tree T' can not be modified, $T = T'$.
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- BOTH: the combination of SUB and ADD.

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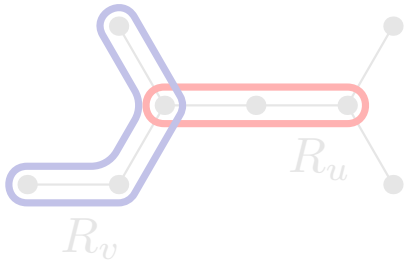

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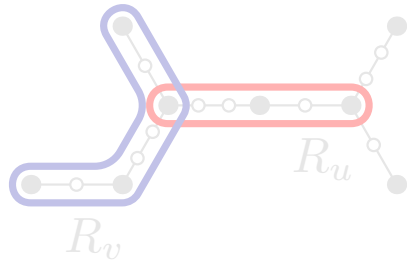

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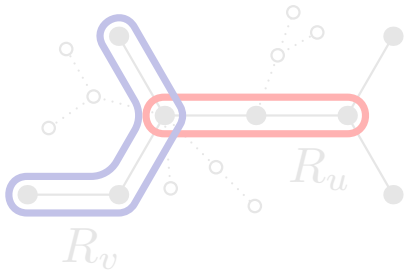

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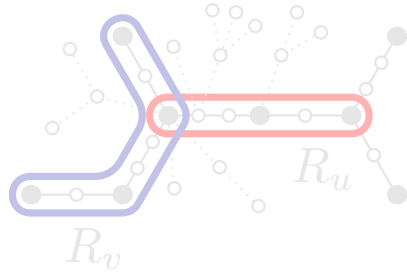

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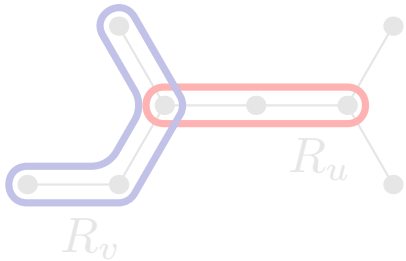

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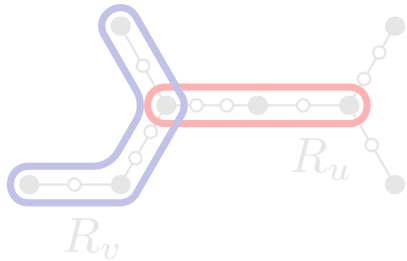

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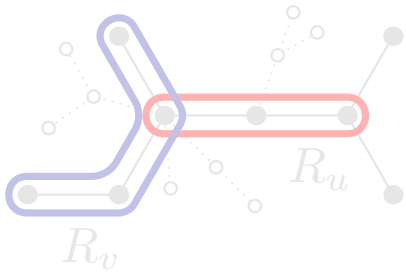

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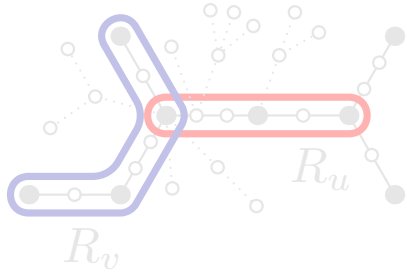

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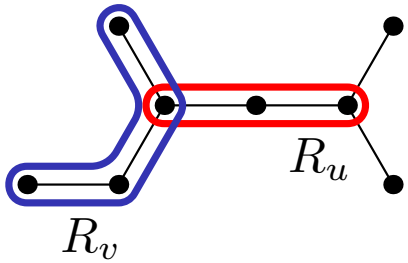
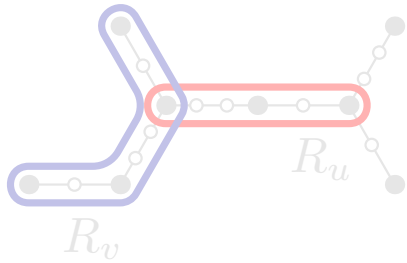
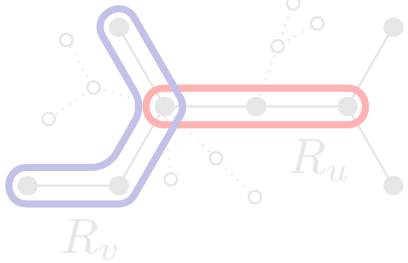


SUB:



ADD:



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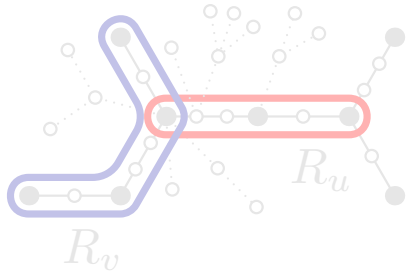

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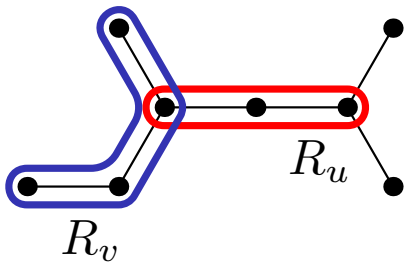

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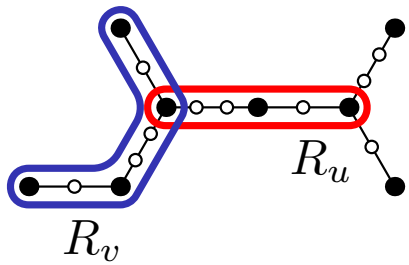

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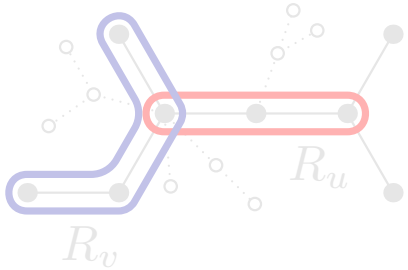

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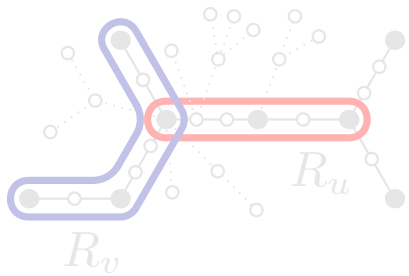

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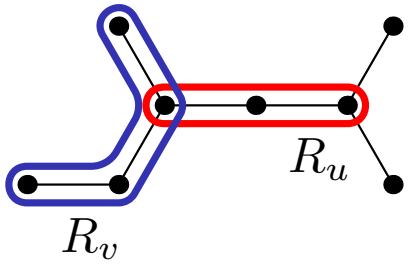

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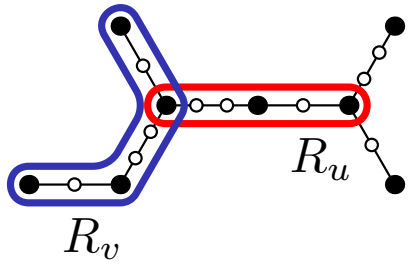

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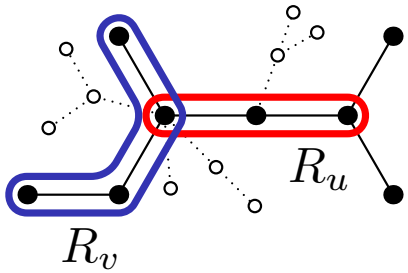

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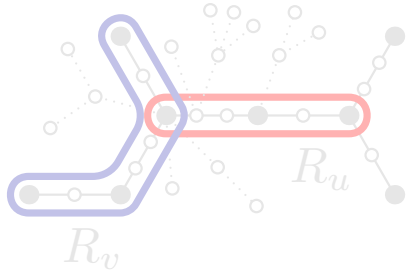

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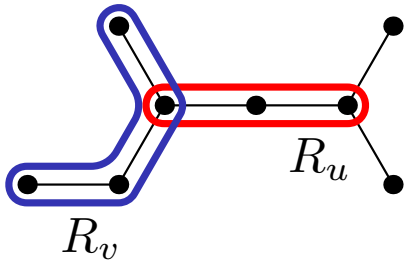

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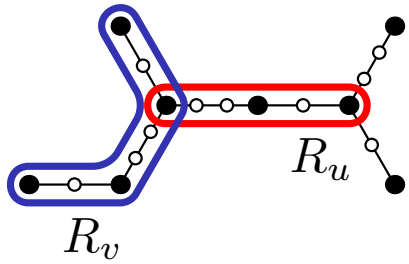

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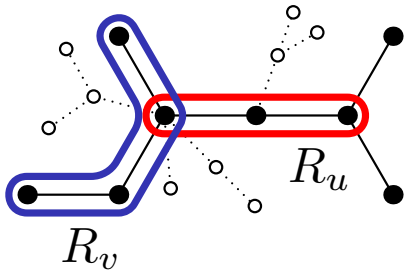

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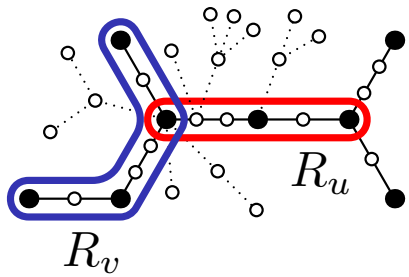

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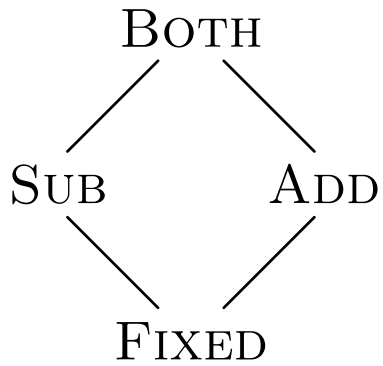

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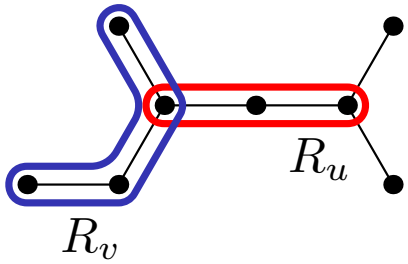

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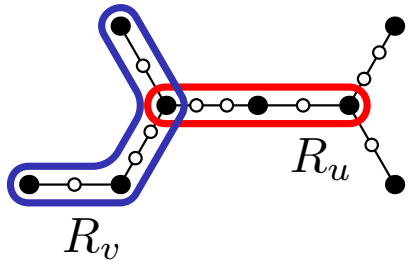

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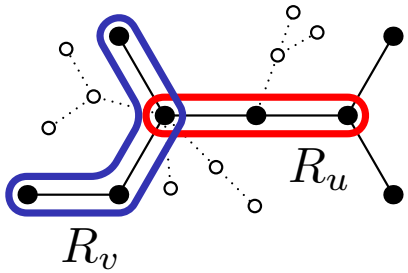

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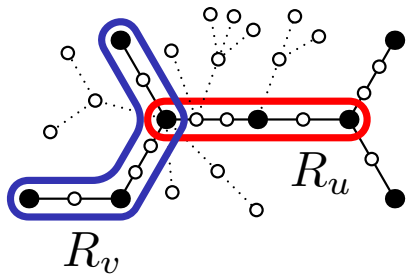

We consider the problems RepExt and Recog* (with $G' = \emptyset$).

		PINT	INT	PATH	CHOR
Fixed	Recog*	$\mathcal{O}(n+m)$	$\mathcal{O}(n+m)$	NP-complete	NP-complete
	RepExt	NP-complete	NP-complete	NP-complete	NP-complete
Sub	Recog*	$\mathcal{O}(n+m)$	$\mathcal{O}(n+m)$	NP-complete	NP-complete
	RepExt	$\mathcal{O}(n+m)$	$\mathcal{O}(n+m)$	NP-complete	NP-complete
Add	Recog*	$\mathcal{O}(n+m)$	$\mathcal{O}(n+m)$	$\mathcal{O}(nm)$	$\mathcal{O}(n+m)$
	RepExt	$\mathcal{O}(n+m)$	NP-complete	NP-complete	NP-complete
Both	Recog*	$\mathcal{O}(n+m)$	$\mathcal{O}(n+m)$	$\mathcal{O}(nm)$	$\mathcal{O}(n+m)$
	RepExt	$\mathcal{O}(n+m)$	$\mathcal{O}(n+m)$	**open**	NP-complete

standard recognition algorithms

previous RepExt results

NP-completeness results

All reductions are from the 3-PARTITION problem:

Problem: 3-PARTITION
Input: Integers k, M and $A_1, \ldots, A_{3k}$ with $\frac{M}{4} < A_i < \frac{M}{2}$.
Output: Is it possible to partition $A_1, \ldots, A_{3k}$ into k triples such that each triple sums to M?

The reduction for REPEXT(INT, FIXED):
Input of 3-PARTITION: $k = 2$, $M = 7$ and A_i's are 2, 2, 2, 2, 3 and 3.

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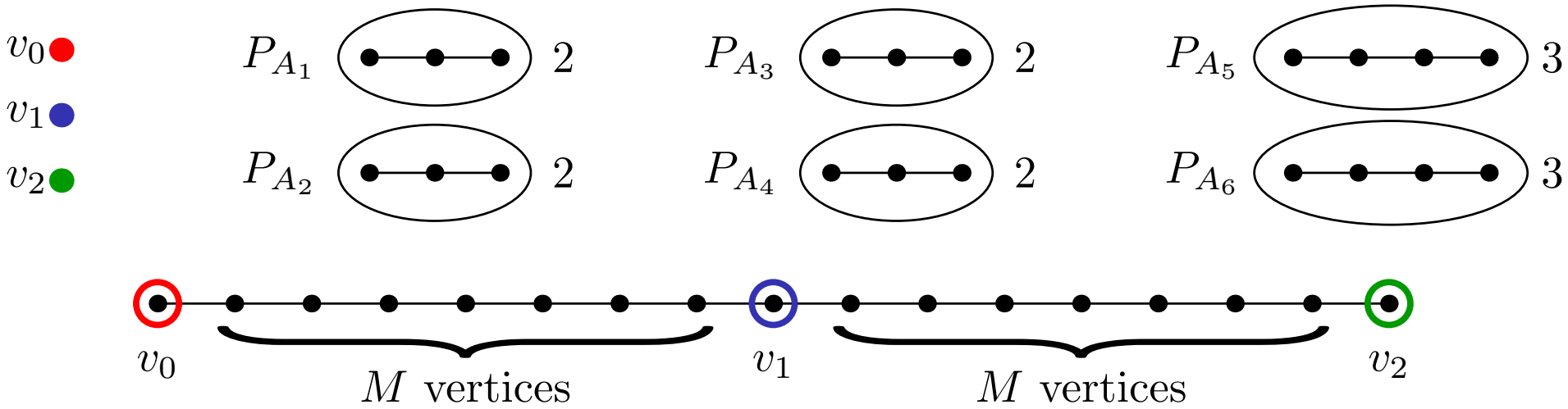

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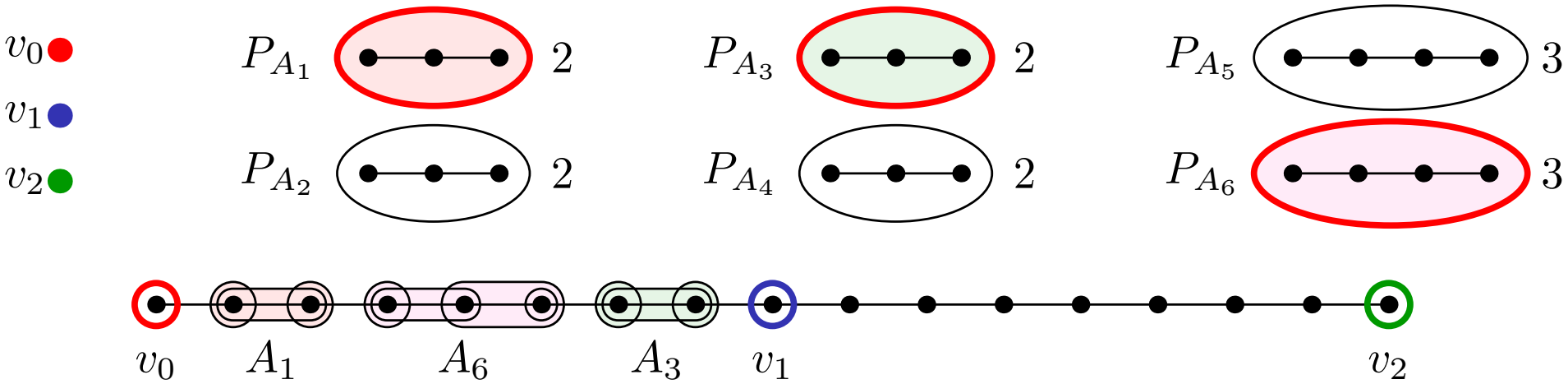

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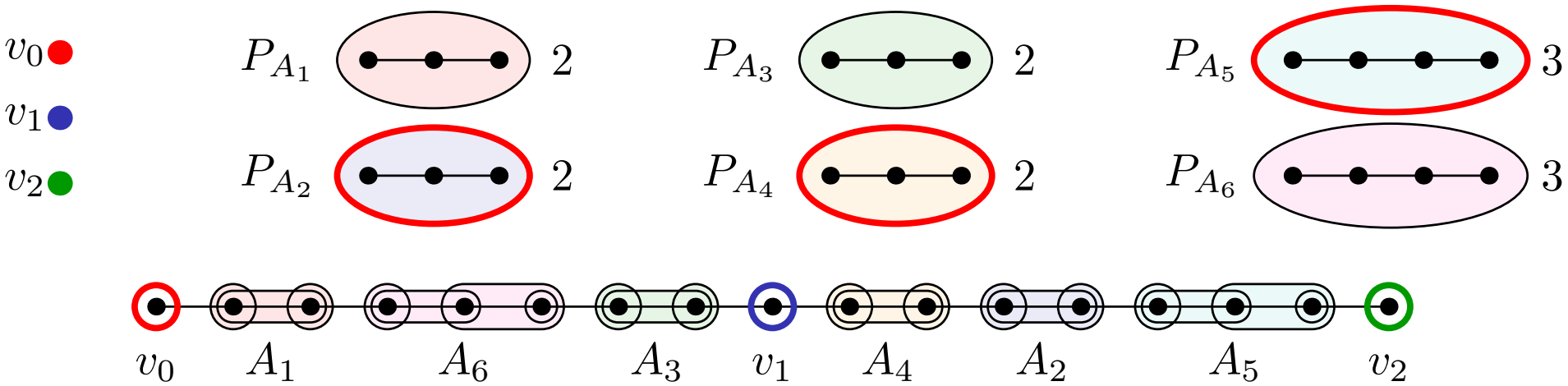
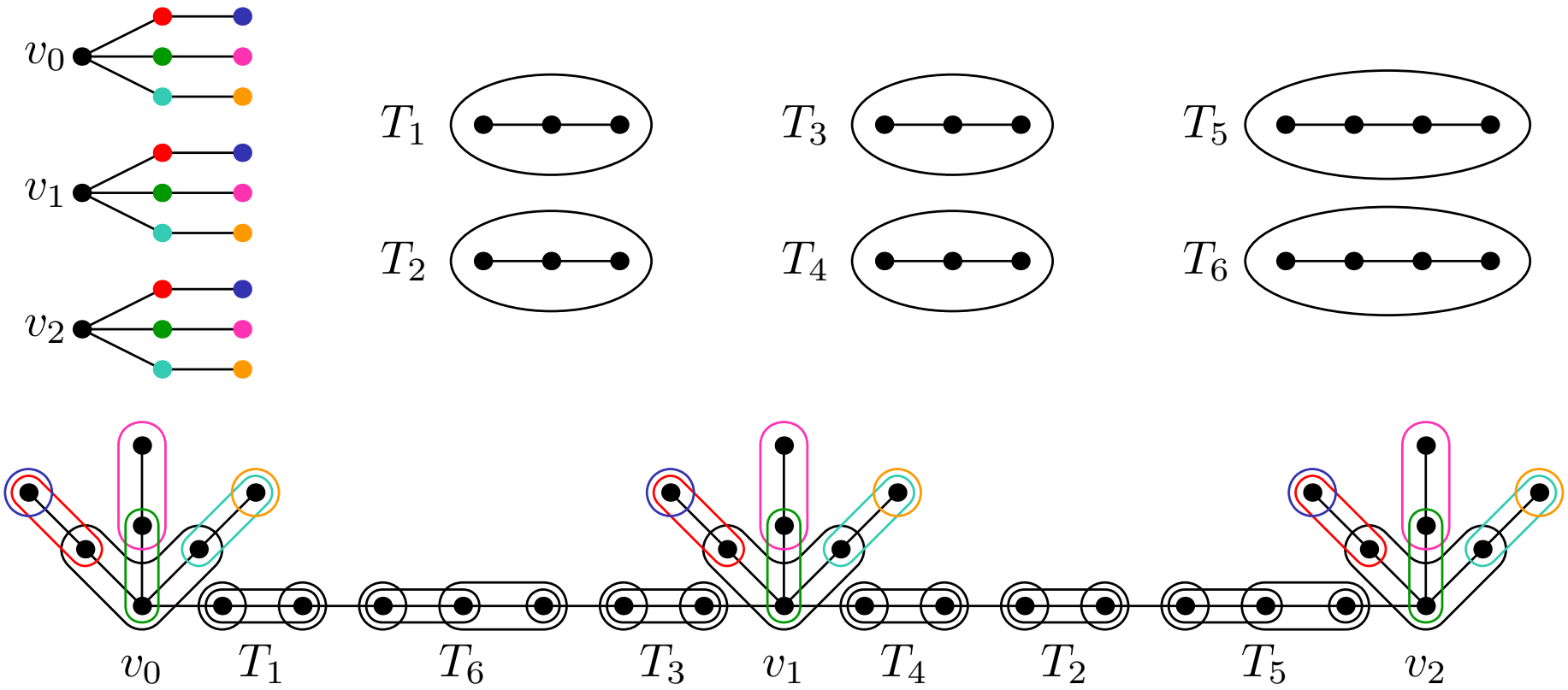
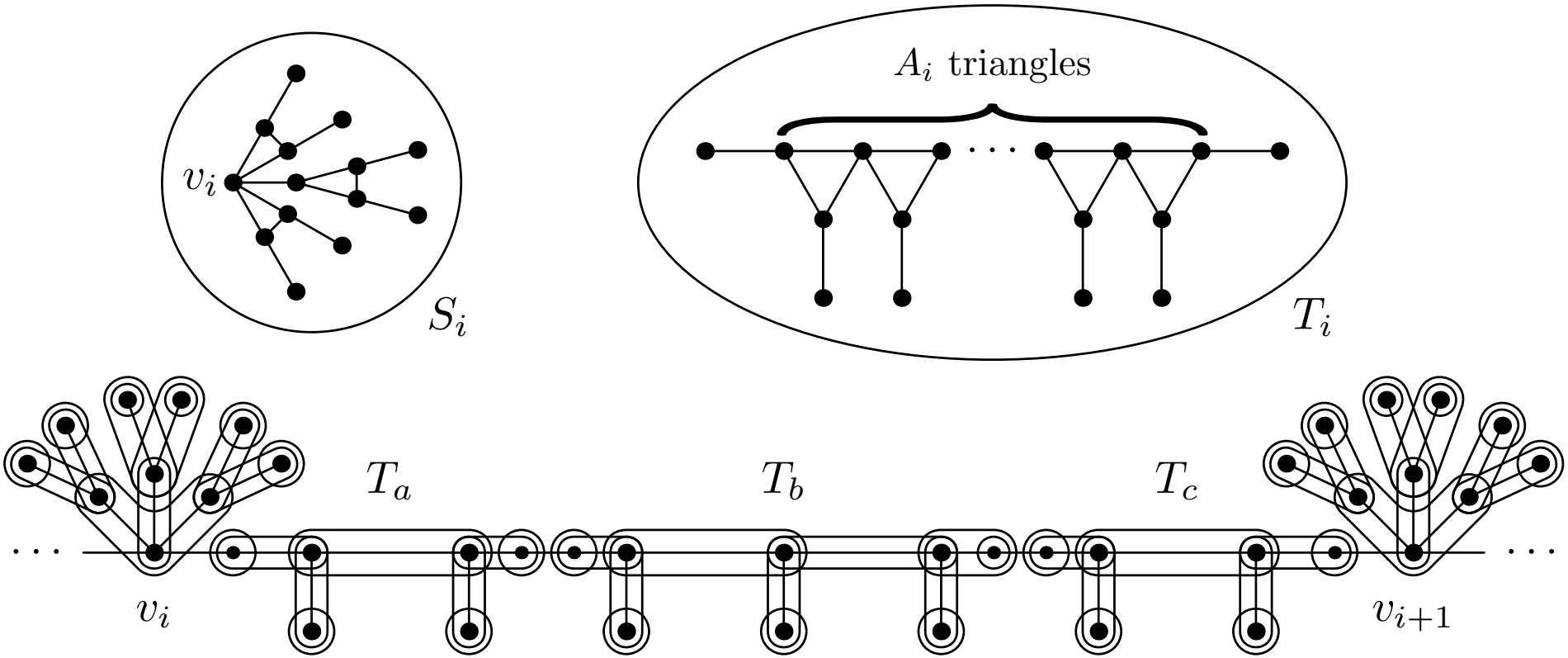


The reduction for RECOG*(PATH, FIXED) and RECOG*(CHOR, FIXED):

Input of 3-PARTITION: $k = 2$, $M = 7$ and A_i's are $2, 2, 2, 2, 3$ and 3.



The reduction for RECOG*(PATH, SUB) and RECOG*(CHOR, SUB):



For ADD type: We have one big pre-drawn subtree covering whole T'.

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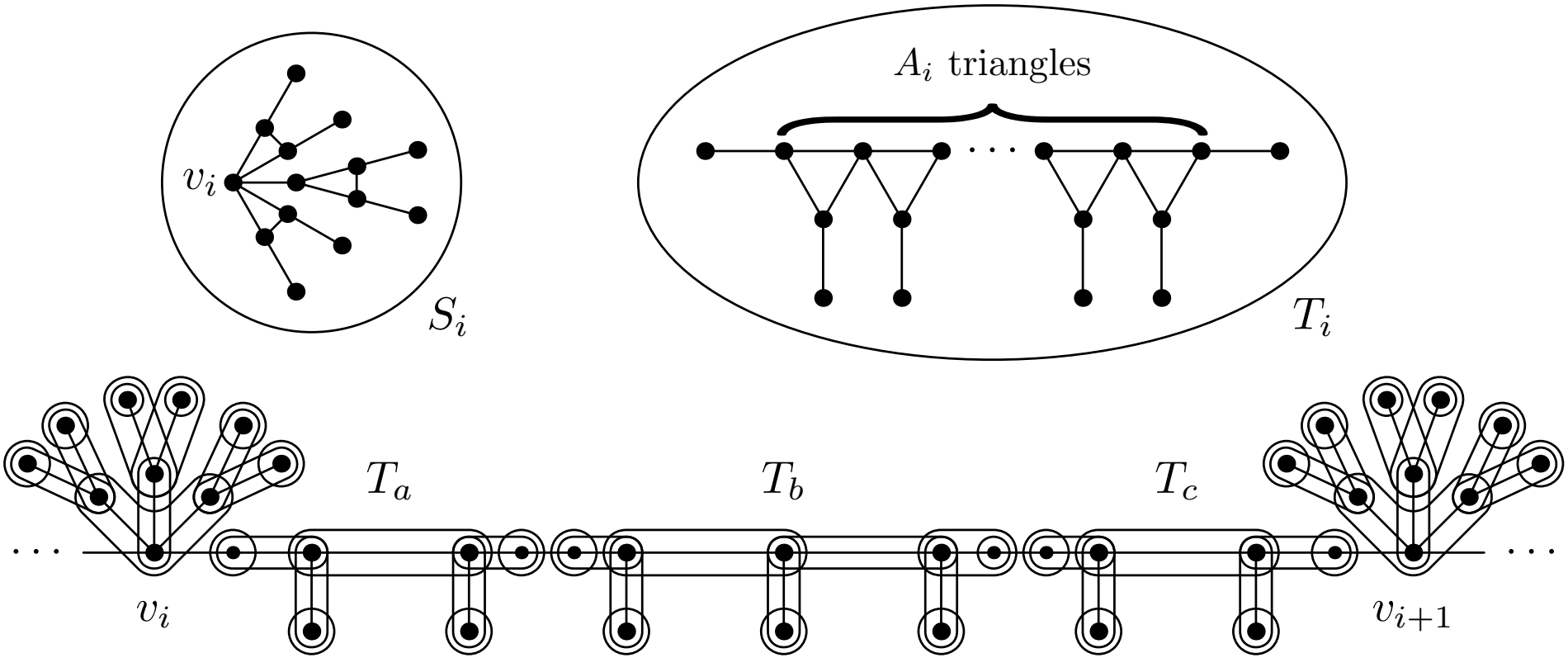

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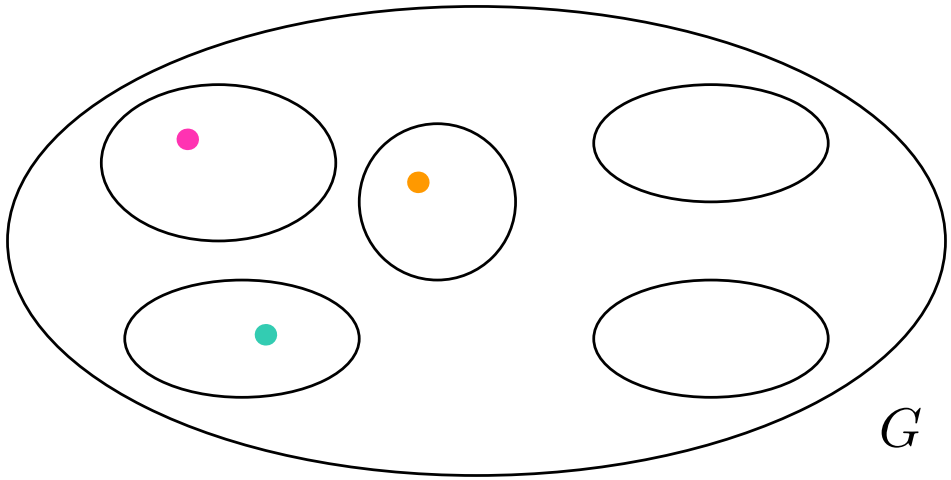
Polynomial results

General ideas for interval graphs:



- There are two types of components: located and unlocated.
- We need to deduce an order of components from left to right.
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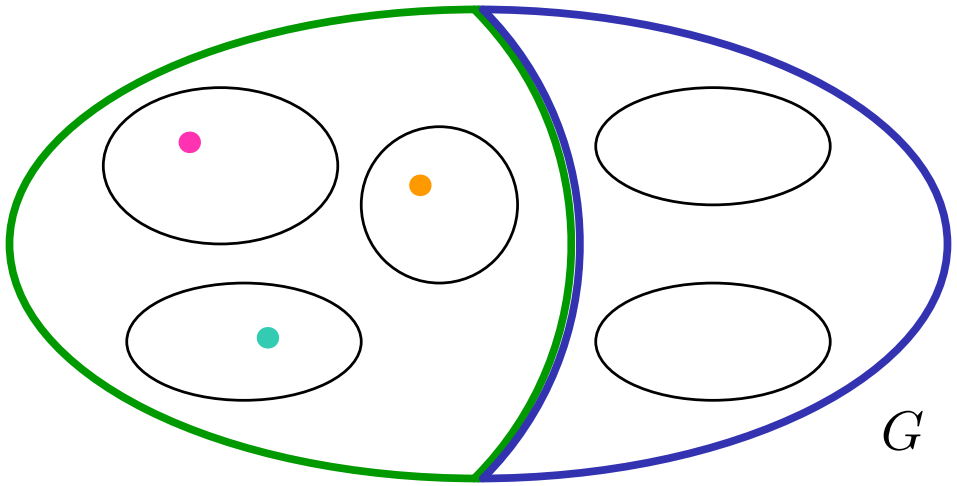

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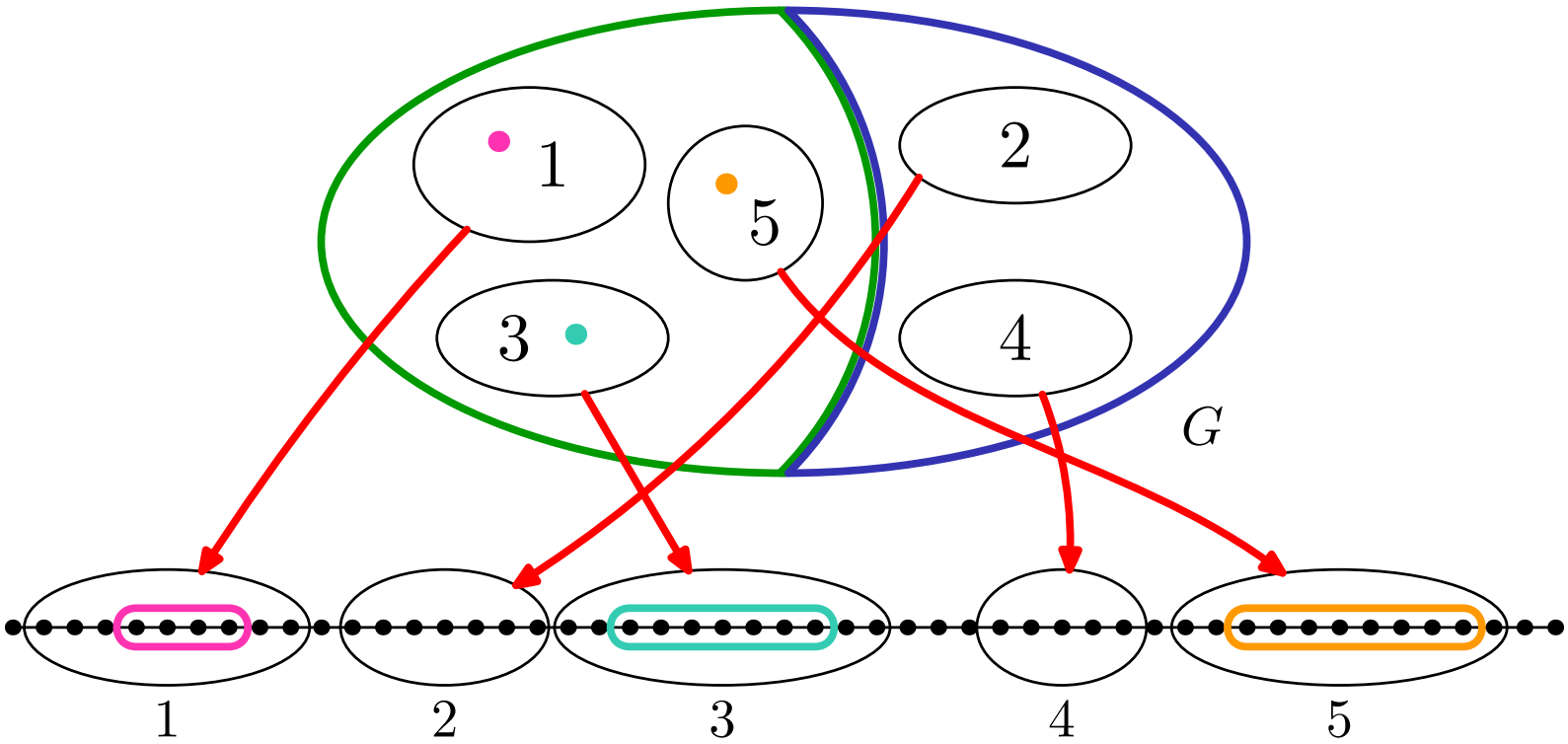

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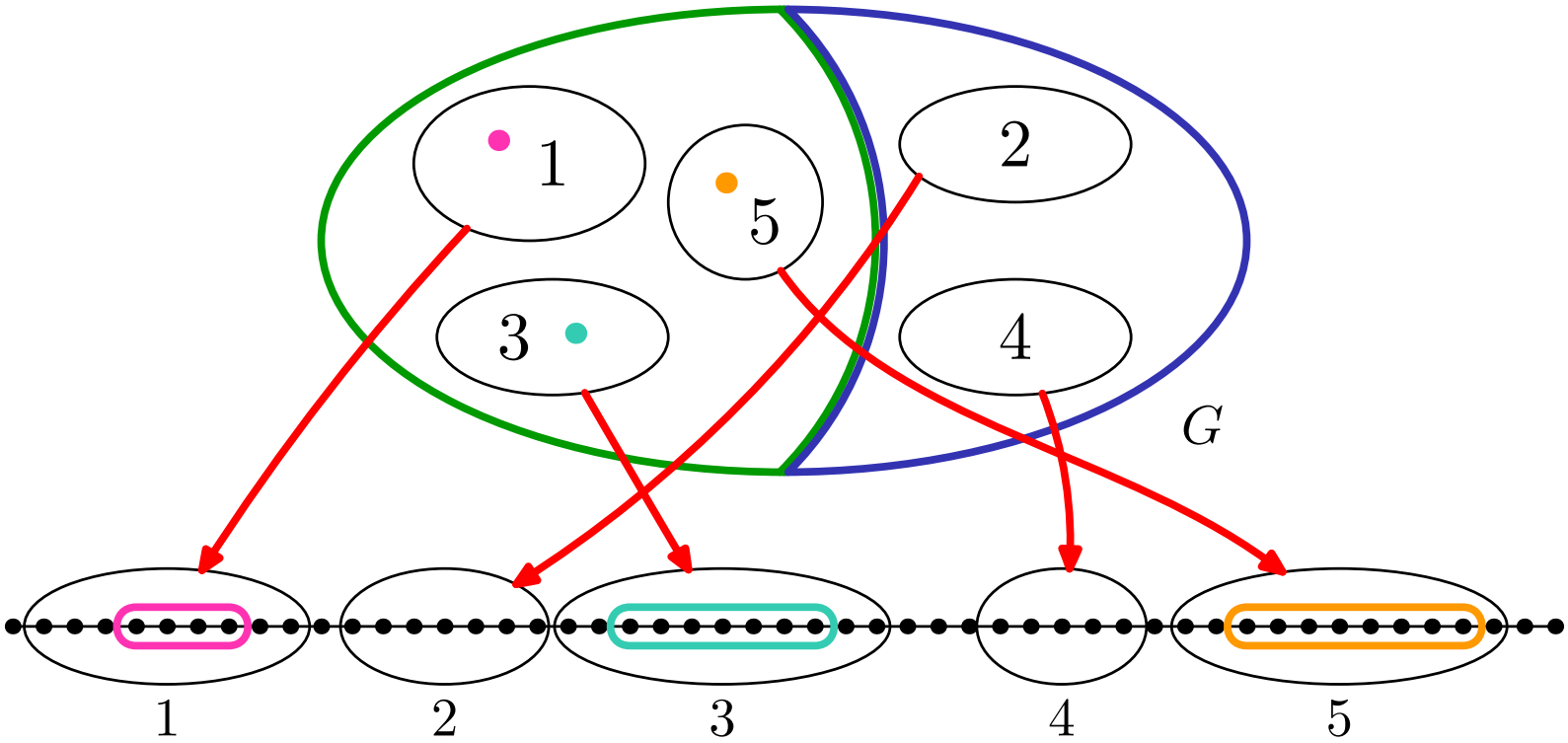

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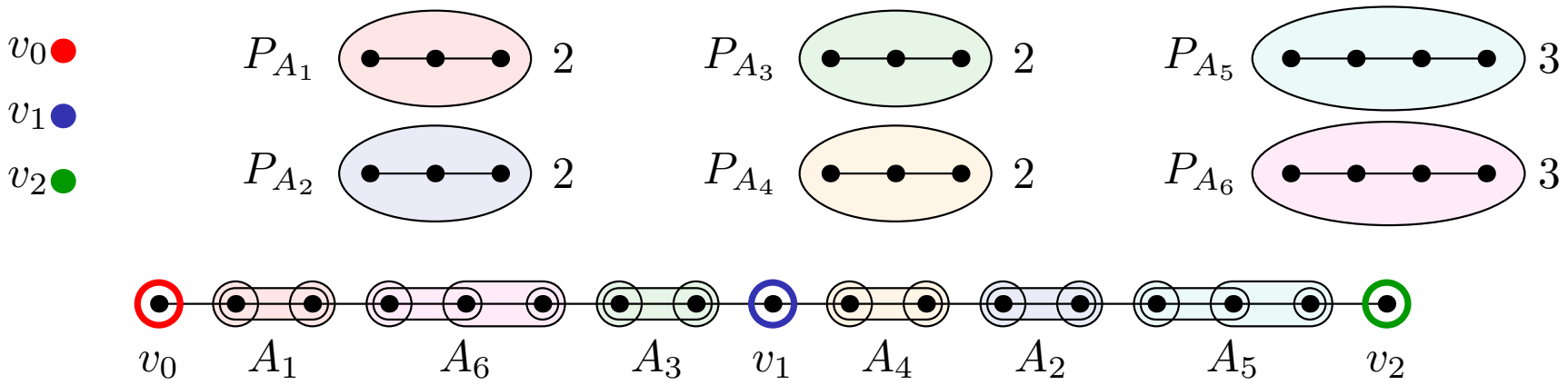

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The problems BINPACKING *and* REPEXT(PINT, FIXED) *are "equivalent":*

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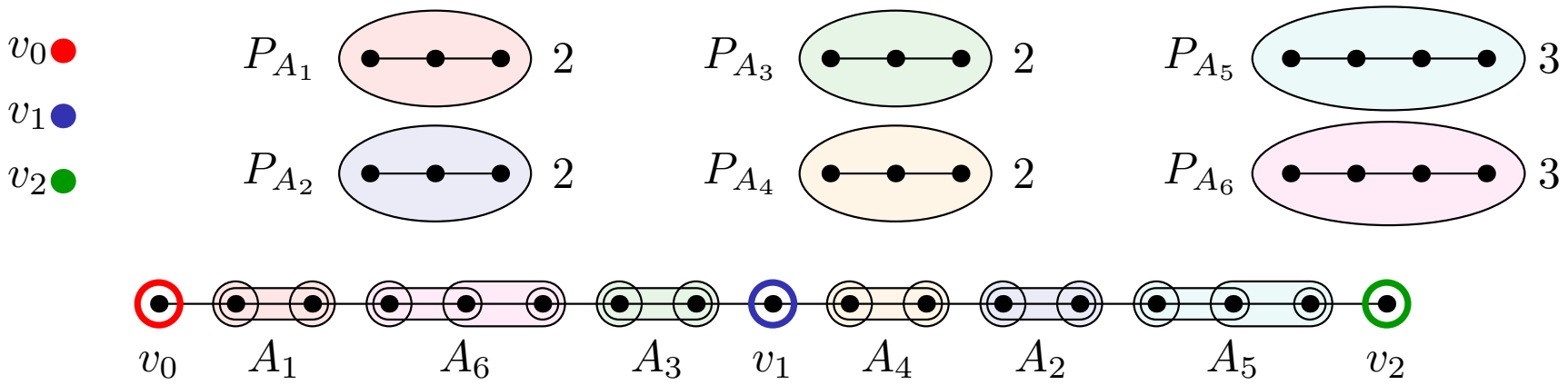

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Open Problem

What is the complexity of RepExt(PATH, Both)?

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How hard is RepExt(INT, Fixed) with respect to the number k of pre-drawn intervals?

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What is the complexity of the RepExt problem for other classes?

CIRCLE, CIRCULAR ARC, ...

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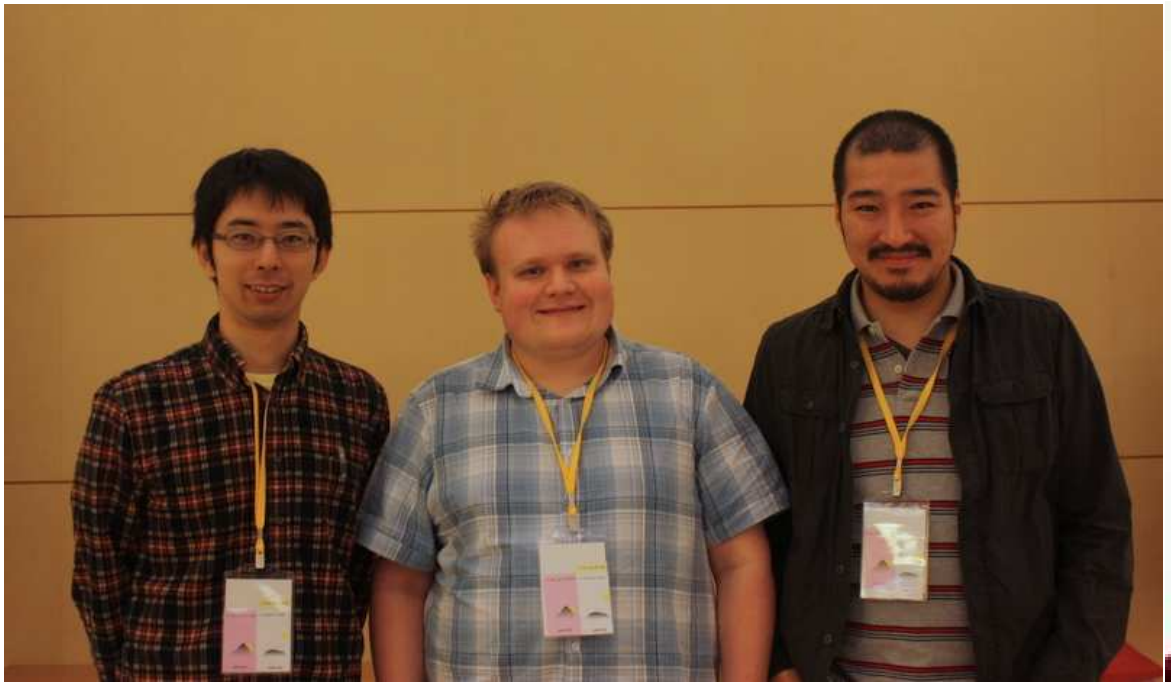

Marry Christmas from Toshiki, Pavel, Yota and Jan.