Bounded Representations of Interval and Proper Interval Graphs

Pavel Klavík

joint work with Martin Balko
and Yota Otachi

ISAAC 2013

Why PROPER INT $\neq$ UNIT INT?

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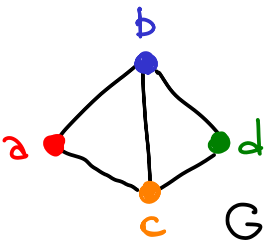
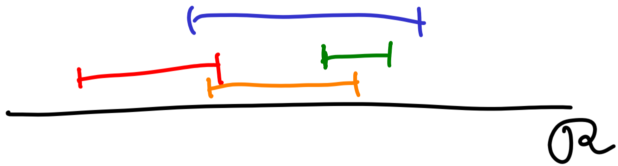
Interval graphs INT

$$\mathcal{R} = \{ I_v \mid v \in V(G) \} \text{ such that}$$

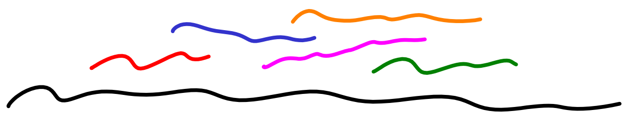
$$I_u \cap I_v \neq \emptyset \iff uv \in E(G)$$



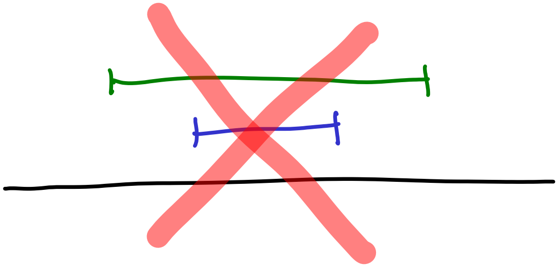


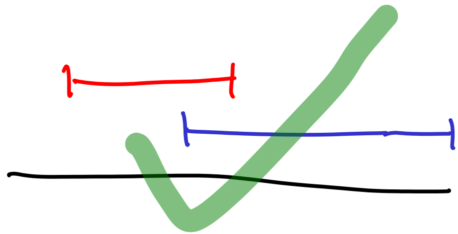
1957 – Hajós

1959 – used for DNA by Benzer

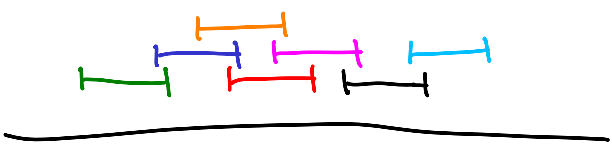


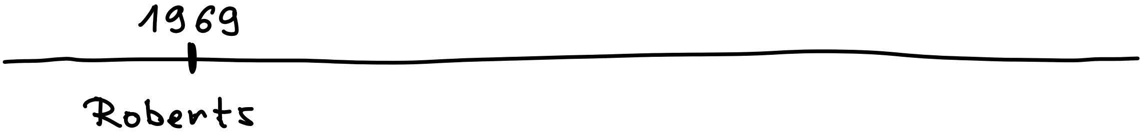
PROPER INT – no interval is a subset of another one



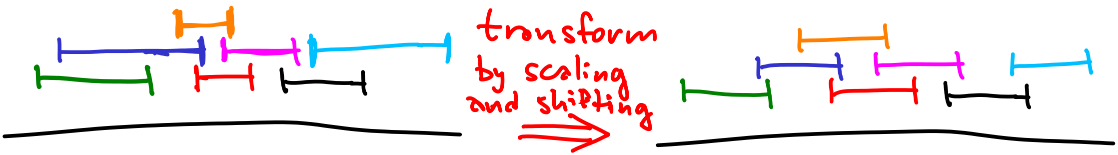


UNIT INT – all intervals are of the unit length





Robert's Theorem: PROPER INT = UNIT INT



Which is great, since proper interval graphs have much simpler structure.

1969

Roberts

What does this equality truly mean?

PROPER INT = UNIT INT

onto mappings

PROPER $\mathcal{R}$ $\supseteq$ UNIT $\mathcal{R}$

topological

geometrical

For $\forall$ PROPER $\mathcal{R}$ there exists some other UNIT $\mathcal{R}$. But is that everything?

1969 2011

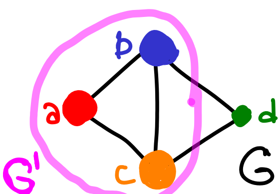
Roberts K., Kratochvíl, Vyskočil

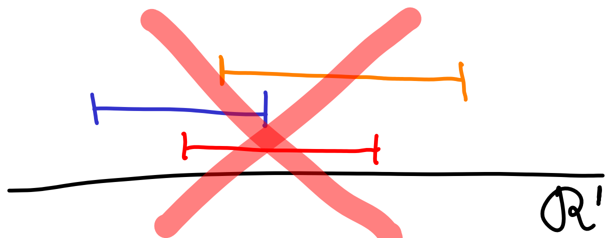
What are the properties of representations, rather than represented graphs?

We studied restricted representation problems.

PARTIAL REPRESENTATION EXTENSION

$$G + \mathcal{R}' \xrightarrow{?} \mathcal{R}$$



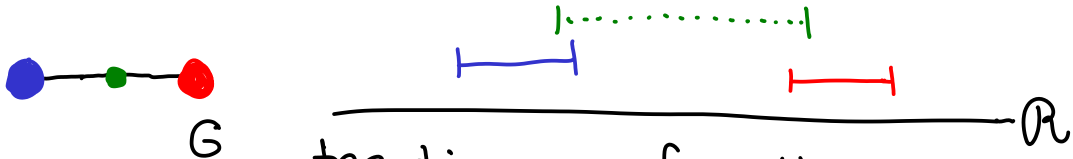


1969 — Roberts

2011 — K., Kratochvíl, Vyskočil

2012 — + Otachi, Rutter, Saitoh, Saumell

REPEXT distinguishes PROPER INT and UNIT INT



too big gap for UNIT INT

We solved REPEXT for both classes:

for PROPER INT $O(n+m)$ — compatible orderings

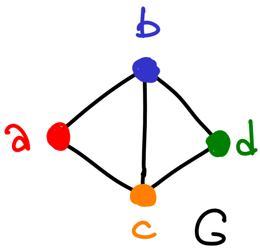
for UNIT INT $\approx O(n^2)$ — additional LP

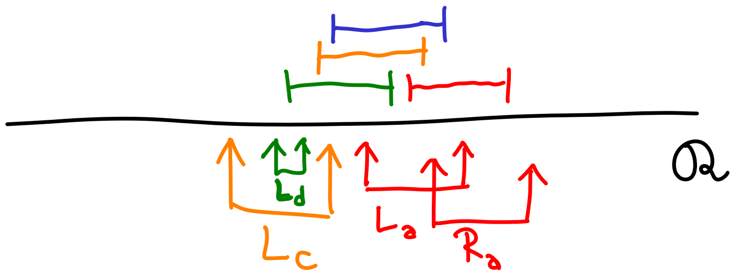
1969 — Roberts

2011 — K., Kratochvíl, Vyskočil

2012 — + Otachi, Rutter, Saitoh, Saumell

Bounded Representation Problem





We require $\ell(I_v) \in L_v,\ r(I_v) \in R_v$

1969 — Roberts

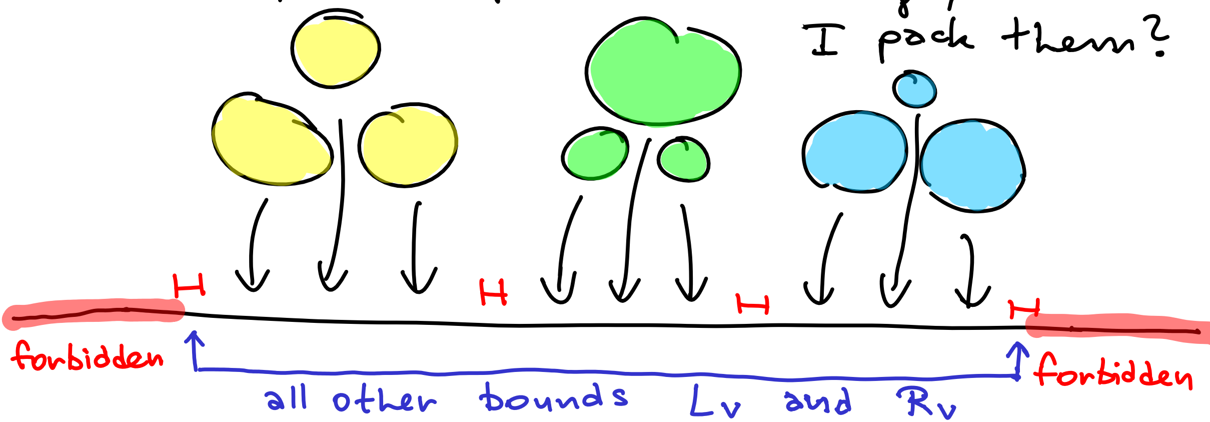
2011 — K., Kratochvíl, Vyskočil

2012 — + Otachi, Rutter, Saitoh, Saumell

Thm: BOUNDREP(UNIT INT) is NP-complete.

Proof: Reduction from 3-PARTITION

Each component represents one integer, can I pack them?

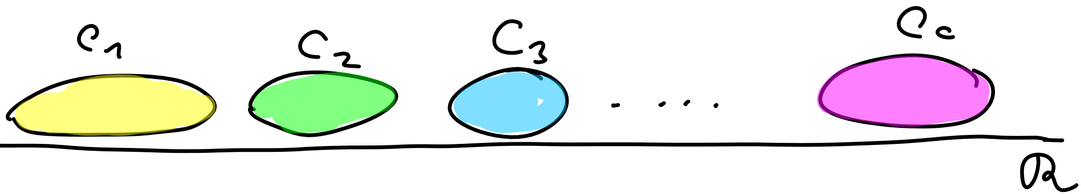


1969 — Roberts

2011 — K., Kratochvíl, Vyskočil

2012 — +Otachi, Rutter, Saitoh, Saumell

Every INT $\mathbb{R}$ has the components ordered from left to right $C_1 \blacktriangleleft C_2 \blacktriangleleft \cdots \blacktriangleleft C_c$:



- If we know $\blacktriangleleft$, we can solve it by LP.
- The reduction is based on limited space.

1969 — Roberts

2011 — K., Kratochvíl, Vyskočil

2012 — +Otachi, Rutter, Saitoh, Saumell

2013 — Balko, K., Otachi

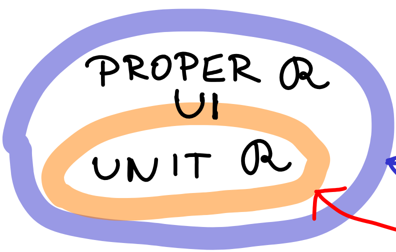
Is there some topological reduction?

No! (Unless P = NP, of course.)

Thm: BoundRep(INT) $O(n+m)$

BoundRep(PROPER INT) $O(n^2)$

$\exists$ problem $\mathcal{P}$ asking for a restricted rep.



polynomially solvable

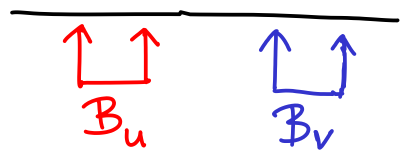
NP-complete

What is the difference for PROPER INT?

We can derive the ordering $\blacktriangleleft$.

The bounds give some partial ordering $\blacktriangleleft'$

If $u \in C$, $v \in C'$ and



$r(B_u) \leq \ell(B_v)$

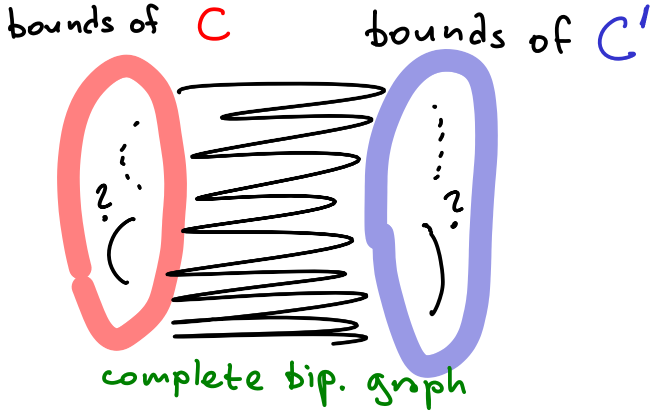
$\Rightarrow$ we put $C \blacktriangleleft' C'$.

And then we choose any linear extension $\blacktriangleleft$

Prop: If there exists any bounded rep., then there exists bounded rep. in $\blacktriangleleft$.

Suppose that we have just two incomparable components C and C'.

Construct a graph A on the bounds

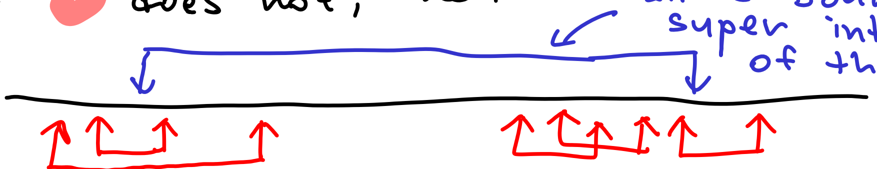


$B_u B_v \in E(A)$ iff they non-trivially intersect.

Either , or  induces a clique.

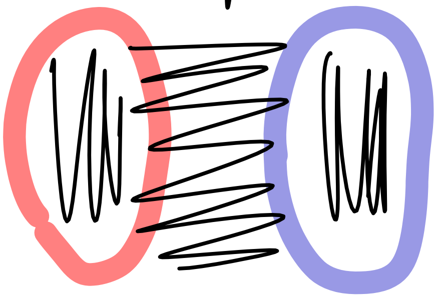
Suppose  does not, then:

all C' bounds super internal of this



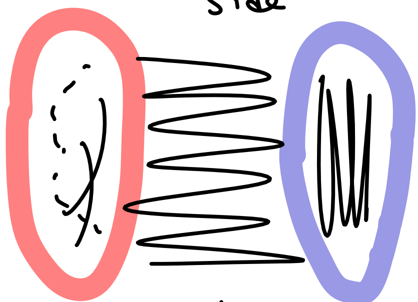
So we get two cases for A:

complete



or

complete on one side

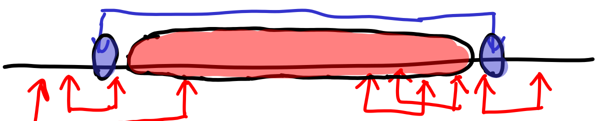


$\Downarrow$

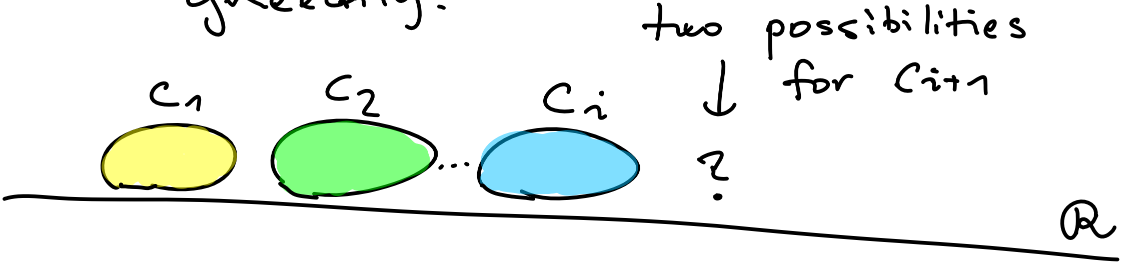
By Helly property $\exists$ ϵ-large segment covered by all bounds.

$\Downarrow$

pack one component into ϵ-large segment to left or right, the other in between.



If we know the ordering ◀,
we proceed components from
left to right and place them
greedily.



Similar to the result for BOUNDREP(UNIT INT)?

A straight-forward implementation runs
in time $O(n^2)$.

Open Problems

1. Can $O(n^2)$ be improved to $O(n+m)$?

2. What is complexity of BoundRep for other graph classes, e.g., circle graphs?

3. What are other structural and algorithmic differences between UNIT $\mathcal{R}$'s and PROPER $\mathcal{R}$'s?

Thank

You!

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