Why PROPER INT is not equal UNIT INT?

Pavel Klavík

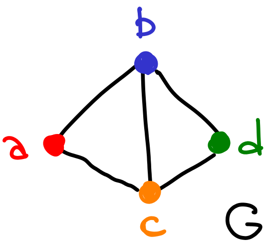
joint work with Martin Balko and Yota Otachi

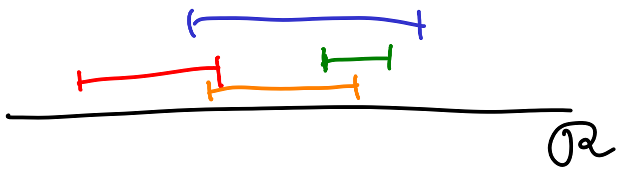
CSGT 2013

Interval graphs INT

$\mathcal{R} = \{ I_v \mid v \in V(G) \}$ such that

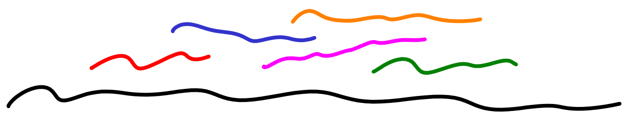
$I_u \cap I_v \neq \emptyset \iff uv \in E(G)$



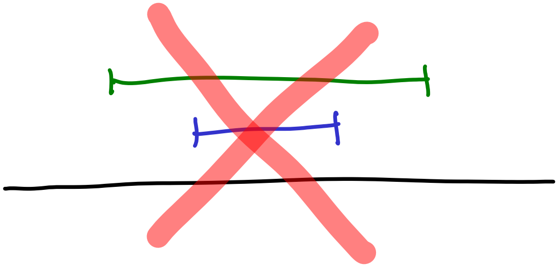


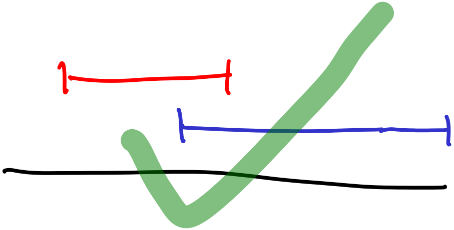
1957 – Hajós

1959 – used for DNA by Benzer

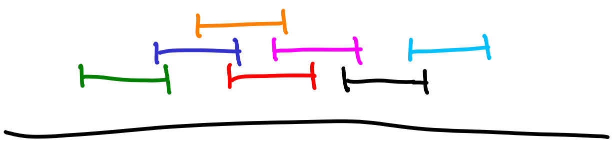


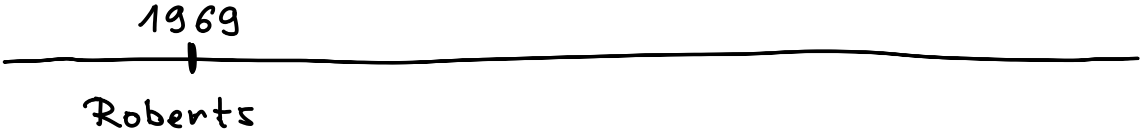
PROPER INT – no interval is a subset of another one



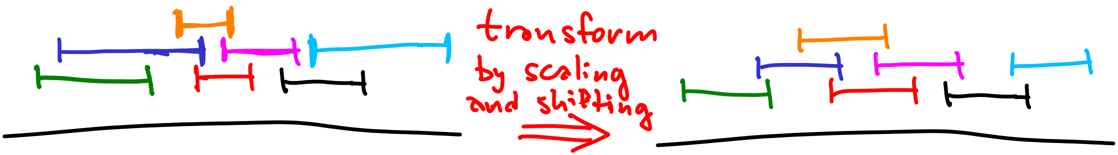


UNIT INT – all intervals are of the unit length

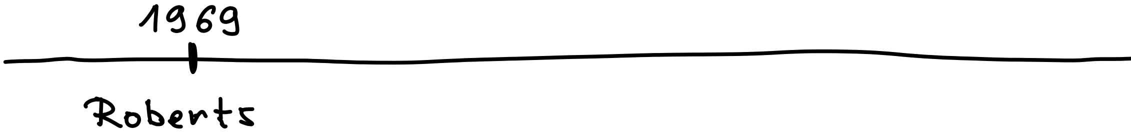




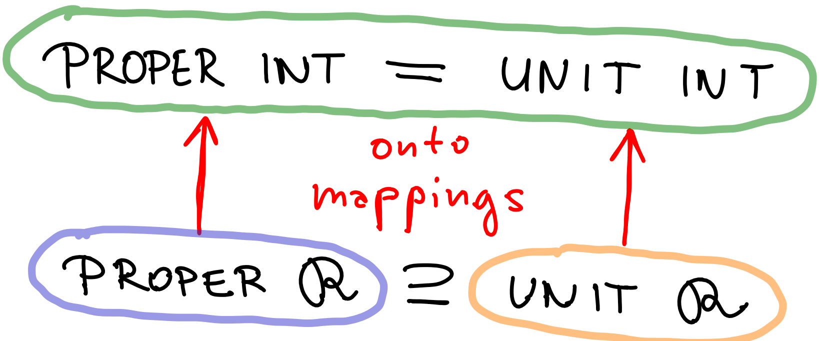
Robert's Theorem: PROPER INT = UNIT INT



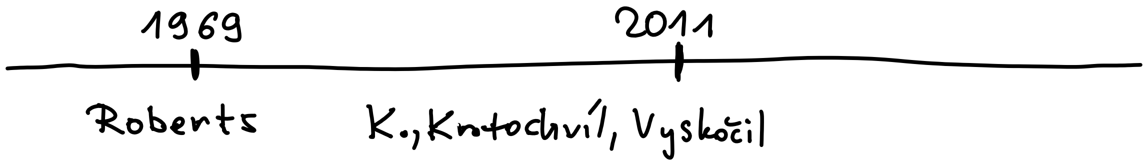
Which is great, since proper interval graphs have much simpler structure.



What does this equality truly mean?



For ∀ PROPER Q there exists some other UNIT Q. But is that everything?

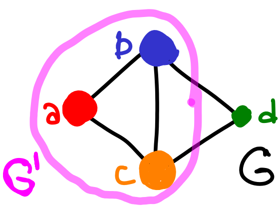


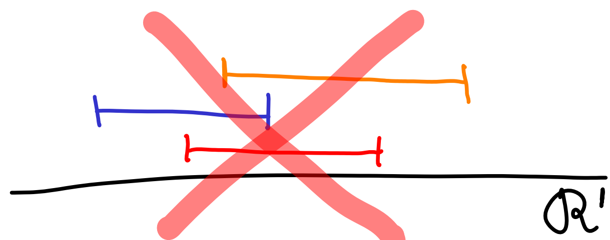
What are the properties of representations, rather than represented graphs?

We studied restricted representation problems.

PARTIAL REPRESENTATION EXTENSION

$$G + \mathcal{R}' \xrightarrow{?} \mathcal{R}$$

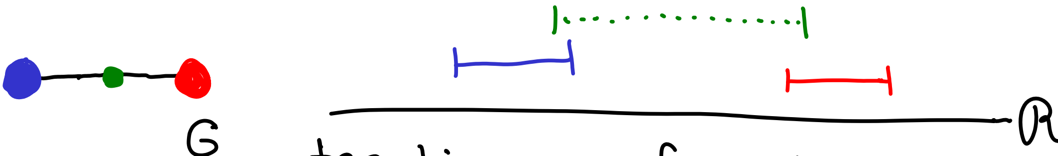




1969 — 2011 — 2012

Roberts — K., Kratochvíl, Vyskočil — + Otachi, Rutter, Saitoh, Saumell

RepExt distinguishes PROPER INT and UNIT INT



too big gap for UNIT INT

We solved RepExt for both classes:

for PROPER INT $O(n+m)$ — compatible orderings

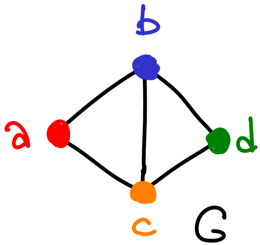
for UNIT INT $\approx O(n^2)$ - additional LP

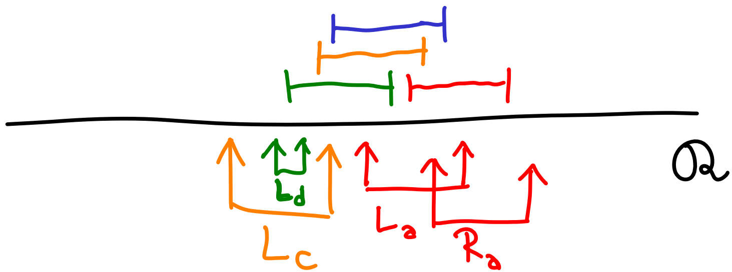
1969 — Roberts

2011 — K., Kratochvíl, Vyskočil

2012 — + Otachi, Rutter, Saitoh, Saumell

Bounded Representation Problem





We require $\ell(I_v) \in L_v,\ r(I_v) \in R_v$

1969 — Roberts

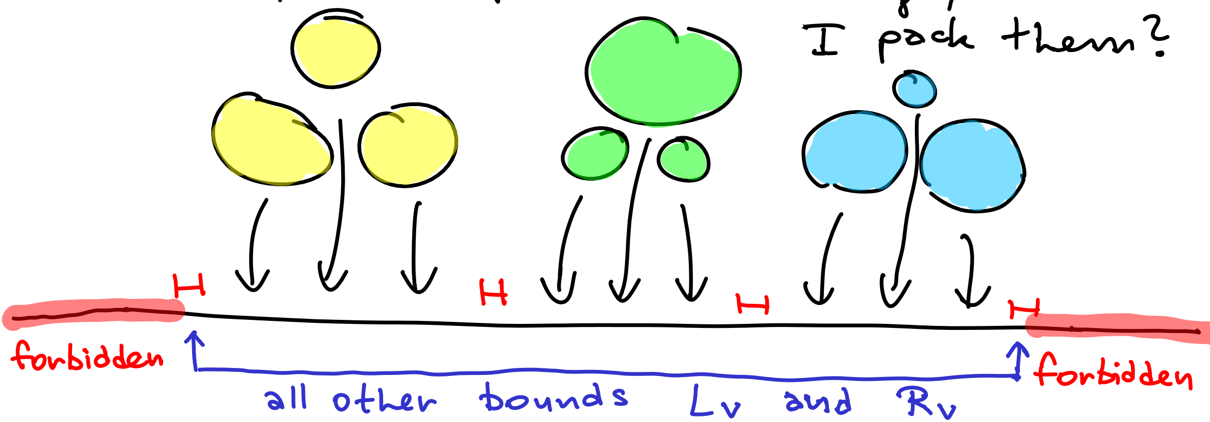
2011 — K., Kratochvíl, Vyskočil

2012 — + Otachi, Rutter, Saitoh, Saumell

Thm: BOUNDREP(UNIT INT) is NP-complete.

Proof: Reduction from 3-PARTITION

Each component represents one integer, can I pack them?

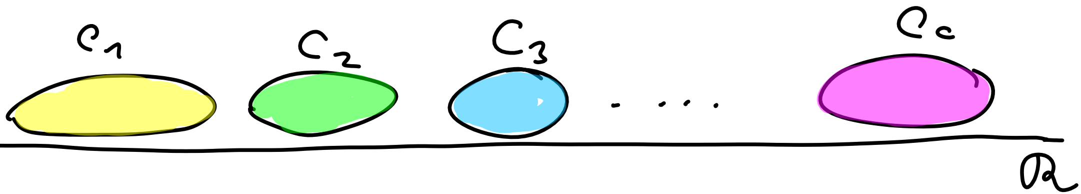


1969 — Roberts

2011 — K., Kratochvíl, Vyskočil

2012 — + Otachi, Rutter, Saitoh, Saumell

Every INT $\mathbb{R}$ has the <u>components ordered from left to right</u> $C_1 \blacktriangleleft C_2 \blacktriangleleft \cdots \blacktriangleleft C_c$:



- If we know $\blacktriangleleft$, we can solve it by LP.
- The reduction is based on limited space.

1969 — Roberts

2011 — K., Kratochvíl, Vyskočil

2012 — + Otachi, Rutter, Saitoh, Saumell

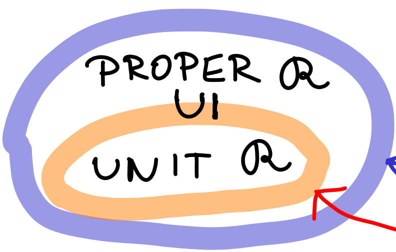
2013 — Balko, K., Otachi

Is there some topological reduction?

No! (Unless P = NP, of course.)

Thm: BoundRep (INT) $O(n+m)$

BoundRep (PROPER INT) $O(n^2)$



∃ problem $\mathcal{P}$ asking for a restricted rep.

← polynomially solvable

← NP-complete

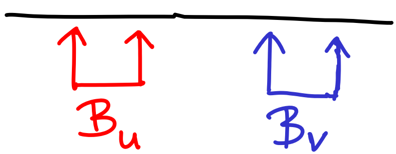
What is the difference for PROPER INT?

We can derive the ordering $\blacktriangleleft$.

The bounds give some partial ordering $\blacktriangleleft'$

If $u \in C$, $v \in C'$ and

$\Rightarrow$ we put $C \blacktriangleleft' C'$.



$r(B_u) \leq \ell(B_v)$

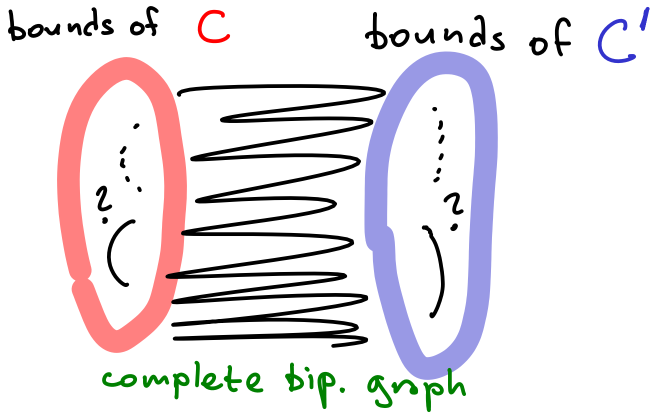
And then we choose any linear extension $\blacktriangleleft$

Prop: If there exists any bounded rep., then there exists bounded rep. in $\blacktriangleleft$.

Suppose that we have just two incomparable components C and C'.

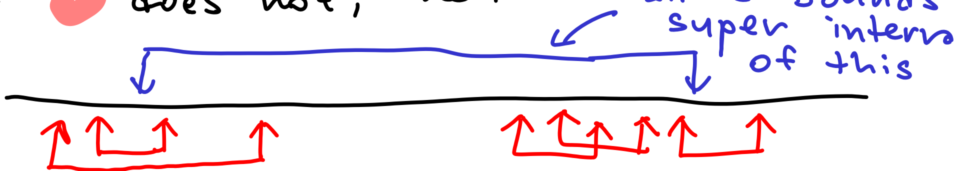
Construct a graph A on the bounds

$B_u B_v \in E(A)$ iff they non-trivially intersect.



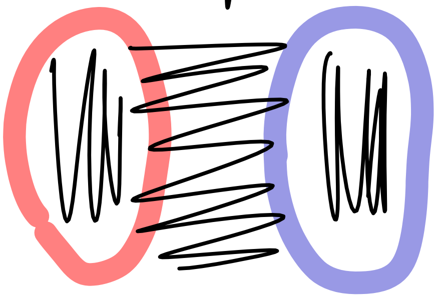
Either , or  induces a clique.

Suppose  does not, then:



So we get two cases for A:

complete

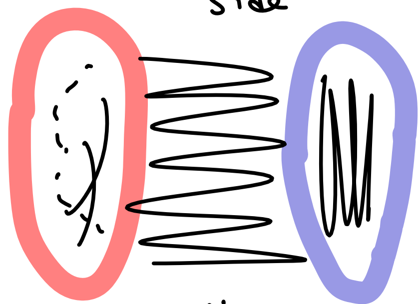


$\Downarrow$

By Helly property $\exists$ ε-large segment covered by all bounds.

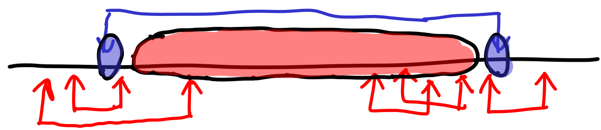
or

complete on one side

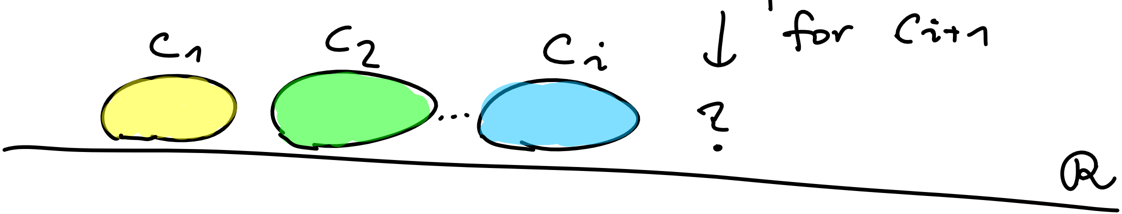


$\Downarrow$

pack one component into ε-large segment to left or right, the other in between.



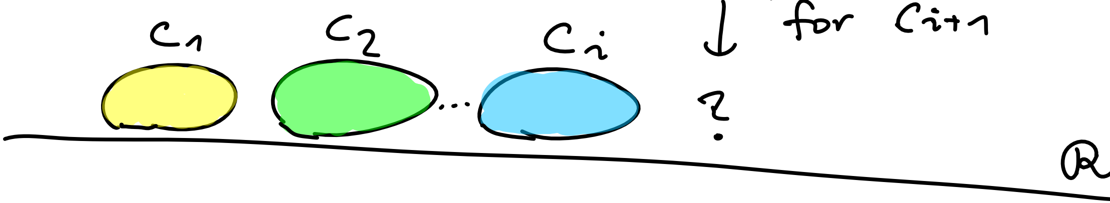
If we know the ordering $\blacktriangleleft$, we proceed components from left to right and place them greedily.



Similar to the result for BOUNDREP(UNIT INT)?

If we know the ordering ◀, we proceed components from left to right and place them greedily.

two possibilities for c_{i+1}



Similar to the result for BOUNDREP(UNIT INT)

Thank You!